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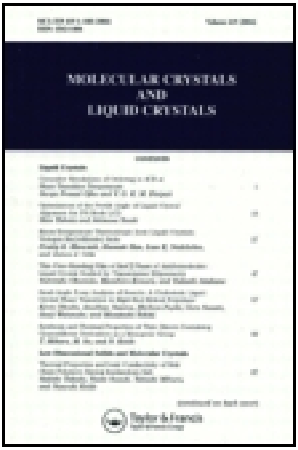

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Light-Induced Frederiks Transition Threshold in Nematic Cell with an Arbitrary Inhomogeneity of the Director anchoring Energy

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The system of algebraic equations which allow to calculate the light induced Frederiks transition threshold in nematic cell with director inhomogeneous anchoring energy is obtained for a general case of light field with spatially modulated intensity. It is found the analytical expressions for threshold in the case of small deviations of anchoring energy from homogeneous value. At the great ones the results of numerical calculations of threshold for some types of the anchoring energy inhomogeneity are presented. The dependence of threshold on the spatial period of light intensity modulation as well as the anchoring energy parameters is discussed.

Keywords: Frederiks transition; anchoring energy; inhomogeneous anchoring

INTRODUCTION

Since the time of experimental revealing of the light induced Frederiks transition (LIFT)[1] the significant attention was spared to in-

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vestigation of the effects of light induced director reorientation in nematic cells[2,3]. Interest to these effects and on the whole to the physics of processes in nematic cells under the action of light fields has grown after the revealing of essential decreasing (on two orders of value) of LIFT threshold in nematics containing small concentration of some impurity molecules[4–6]. It was studied the influence on LIFT threshold of the molecular light-induced conformational transformations as well as the spatial period value of light intensity modulation[7]. The last problem arises in connection with the possibility of recording in nematic cells of the dynamic holographic gratings using the orientational or conformational nonlinearity[8]. Despite of the fact that light induced director reorientation is a bulk effect, its main parameters such as a degree of the director reorientation or the threshold value depend essentially on the interaction of the nematic liquid crystal (NLC) with the cell surface. The influence of the cell surface can be so significant that it provokes the spontaneous Frederiks transition[9,10].

The director anchoring energy with the cell surface is one of the most important parameters determining the conditions for director on the cell surface. Usually it introduces as some phenomenological quantity which at the theoretical calculations is assumed constant (infinite or finite) on all the cell surface.

In present paper we consider the nematic cell with arbitrary dependence of the director anchoring energy on the coordinate on the cell surface. Geometry of the incident light field is supposed the same as at the recording of the dynamical holographic gratings. In this case the interference distribution of the light intensity with given spatial period is created in the nematic cell bulk by two incident coherent light waves. It is considered the dependence of LIFT threshold in this light field on the inhomogeneous part of the nematic director anchoring energy.

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EQUATION FOR THE DIRECTOR

Let we have the cell of homeotropically aligned NLC with an initial (undisturbed) orientation of the director $\vec{n}$ along the z axis which is bounded by the planes $z = 0$ and $z = L$. We assume that two plane monochromatic light waves are incident on the cell making the angle 2θ between their wave vectors of value q. The waves have equal amplitudes E_0 and are polarized along the x axis but their wave vectors are oriented symmetrically with respect to the xz plane.

In consequence of interference in the cell bulk the electric vector of the light wave field takes the form

$$\vec{E} = \frac{1}{2}\left[\vec{E}(\vec{r})\exp(-i\omega t) + \vec{E}^*(\vec{r})\exp(i\omega t)\right],$$

$$\vec{E}(\vec{r}) = 2\vec{E}_0\cos\left(\frac{1}{2}\Delta qy\right)\exp(iqz\cos\theta), \quad \Delta q = 2q\sin\theta, \qquad (1)$$

that is the light intensity grating with spatial period $2\pi/\Delta q$ is creatied in the cell bulk.

We assume now that the director anchoring energy with the cell planes is inhomogeneous, namely, is an arbitrary function of the coordinate y (of course, this function has to be smooth enough to use the continual theory of the NLC alignment). One can present it in the following form

$$W(y) = W + \alpha(y), \qquad (2)$$

where the constant $W > 0$ and $W(y) > 0$ as well.

The surface free energy F_S is taken in the form of Rapini potential[11] but generalized on the considered case of the inhomogeneous anchoring energy:

$$F_S = -\frac{1}{2}\int W(y)(\vec{n}\vec{e})^2\,dS, \qquad (3)$$

where $\vec{e}$ is a unit vector along the director easy orientation axis (the z axis).

Minimizing the nematic cell free energy in the electric field (1) in the one constant approximation we obtain the equation for the

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director angle $\varphi(y,z)$ with the z axis and the boundary conditions. To obtain the threshold value of the light wave electric field it is sufficient to keep in the variational equation only the terms linear in $\varphi(y,z)$. Then one can get the differential equation and the boundary conditions for the director angle $\varphi(y,z)$ in the next form:

$$\hat{L}\varphi(y,z) = 0,$$
$$\hat{L} = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\varepsilon_a \varepsilon_\perp}{2\pi\varepsilon_\| K} E_0^2 \cos^2\left(\frac{1}{2}\Delta q y\right), \tag{4}$$

$$\left(K\frac{\partial\varphi}{\partial z} \mp (W + \alpha(y))\varphi\right)_{z=0,L} = 0. \tag{5}$$

Here K is the Frank constant, $\varepsilon_a = \varepsilon_\| - \varepsilon_\perp$, $\varepsilon_\|$, $\varepsilon_\perp$ are the main values of dielectric susceptibility tensor. Below we consider the solution of the problem (4), (5).

EQUATIONS FOR THRESHOLD

The solution of the equation (4) one can seek in the form of superposition of the operator $\hat{L}$ eigenfunctions. Sobving the equation for eigenfunctions $\hat{L}\Phi_\mu(y,z) = \mu\Phi_\mu(y,z)$ by the method of separation of variables we get

$$\Phi_\mu(y,z) = \sum_\lambda \Phi_\mu^\lambda(y,z), \tag{6}$$

where

$$\Phi_\mu^\lambda(y,z) = (A_\lambda \cos\lambda z + B_\lambda \sin\lambda z)\times$$
$$\times \sum_{m=0}^{\infty} [C_m^{\mu+\lambda}\cos(m\Delta q y) + D_m^{\mu+\lambda}\sin(m\Delta q y)]. \tag{7}$$

Here $\Phi_\mu^\lambda(y,z)$ is a partial solution of the equation $\hat{L}\Phi_\mu = \mu\Phi_\mu$ which corresponds to the fixed value of the separation parameter λ: A_λ, B_λ are the arbitrary constants which depend on the value λ; $C_m^{\mu+\lambda}$, $D_m^{\mu+\lambda}$ are the constants in the Mathieu functions. One can demand the

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linear combination of the functions (7) corresponding to the different eigenvalue μ, namely,

$$\Phi^{\lambda}(y,z) = \sum_{\mu} \Phi^{\lambda}_{\mu}(y,z) = (A_{\lambda}\cos\lambda z + B_{\lambda}\sin\lambda z)\times \\ \times \sum_{m=0}^{\infty} [C^{\lambda}_{m}\cos(m\Delta qy) + D^{\lambda}_{m}\sin(m\Delta qy)], \tag{8}$$

where $C^{\lambda}_{m} = \sum_{\mu} C^{\mu+\lambda}_{m}$, $D^{\lambda}_{m} = \sum_{\mu} D^{\mu+\lambda}_{m}$, to satisfy the boundary conditions (5). As a result one can get the following system of the homogeneous algebraic equations for the unknown quantities $\tilde{A}^{\lambda}_{m} = A_{\lambda}C^{\lambda}_{m}$, $\tilde{B}^{\lambda}_{m} = B_{\lambda}C^{\lambda}_{m}$, $\tilde{C}^{\lambda}_{m} = A_{\lambda}D^{\lambda}_{m}$, $\tilde{D}^{\lambda}_{m} = B_{\lambda}D^{\lambda}_{m}$:

$$\begin{aligned}
&-\lambda\sin\lambda L\cdot\tilde{A}^{\lambda}_{m} + \left(2\lambda\cos\lambda L + \frac{W}{K}\sin\lambda L\right)\tilde{B}^{\lambda}_{m} + \\
&+\frac{1}{K}\sum_{l=0}^{\infty}\alpha^{cc}_{ml}\tilde{B}^{\lambda}_{l}\sin\lambda L + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{cs}_{ml}\tilde{D}^{\lambda}_{l}\sin\lambda L = 0,\\
&-\lambda\sin\lambda L\cdot\tilde{C}^{\lambda}_{m} + \left(2\lambda\cos\lambda L + \frac{W}{K}\sin\lambda L\right)\tilde{D}^{\lambda}_{m} + \\
&+\frac{1}{K}\sum_{l=0}^{\infty}\alpha^{sc}_{ml}\tilde{B}^{\lambda}_{l}\sin\lambda L + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{ss}_{ml}\tilde{D}^{\lambda}_{l}\sin\lambda L = 0,\\
&\frac{W}{K}\tilde{A}^{\lambda}_{m} - \lambda\tilde{B}^{\lambda}_{m} + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{cc}_{ml}\tilde{A}^{\lambda}_{l} + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{cs}_{ml}\tilde{C}^{\lambda}_{l} = 0,\\
&\frac{W}{K}\tilde{C}^{\lambda}_{m} - \lambda\tilde{D}^{\lambda}_{m} + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{sc}_{ml}\tilde{A}^{\lambda}_{l} + \frac{1}{K}\sum_{l=0}^{\infty}\alpha^{ss}_{ml}\tilde{C}^{\lambda}_{l} = 0.
\end{aligned} \tag{9}$$

Where

$$\alpha^{cc}_{ml} = \frac{1}{T}\left(1 - \frac{1}{2}\delta_{m,0}\right)\int_{-T}^{T}\alpha(y)\cos(m\Delta qy)\cos(l\Delta qy)\,dy$$

and α^{cs}_{ml}, α^{sc}_{ml}, α^{ss}_{ml} can be obtained replacing in α^{cc}_{ml} respectively cos by sin, $2T$ is a spatial period of the problem. The condition of non-trivial solution of the system of homogeneous equations (9) gives us the equation for the quantity λ. For convenience of the subsequent references we designate this equation as

$$f\left(\lambda, \frac{W}{K}, \frac{\alpha_{ml}}{K}\right) = 0. \tag{10}$$

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The solutions of equation (10) we denote by λ_n where the integer n numbers the solutions. Now the general solution of equation (4) which satisfies the boundary conditions (5) takes the form:

$$\begin{aligned}\varphi(y,z) &= \sum_\lambda \Phi^\lambda(y,z) = \sum_\lambda \sum_\mu \Phi^\lambda_\mu(y,z) = \\ &= \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} [C^n_m \cos(m\Delta qy) + D^n_m \sin(m\Delta qy)] Z_n(z),\end{aligned} \tag{11}$$

where

$$Z_n(z) = A_n \cos\lambda_n z + B_n \sin\lambda_n z. \tag{12}$$

On substitution of the series (11) into the equation (4) and the subsequent using of the orthogonality of the functions $Z_n(z)$ on the interval $[0, L]$ at the different values of n (as the solutions of the Shtourm's problem) as well as the orthogonality of the functions $\cos(m\Delta qy)$, $\sin(m\Delta qy)$ we obtain two separate infinite systems of the homogeneous algebraic equations for the coefficients C^n_m and D^n_m, respectively:

$$\begin{aligned}&a^0_n C^n_0 - p^0 C^n_1 = 0, \quad (a^0_n - 4)C^n_1 - p^0(2C^n_0 + C^n_2) = 0, \\ &(a^0_n - 4m^2)C^n_m - p^0(C^n_{m-1} + C^n_{m+1}) = 0, \qquad m \geq 2\end{aligned} \tag{13}$$

$$\begin{aligned}&(a^0_n - 4)D^n_1 - p^0 D^n_2 = 0, \\ &(a^0_n - 4m^2)D^n_m - p^0(D^n_{m-1} + D^n_{m+1}) = 0, \qquad m \geq 2\end{aligned} \tag{14}$$

where

$$p^0 = -\frac{\varepsilon_a \varepsilon_\perp}{2\pi\varepsilon_\parallel} \frac{E_0^2}{K(\Delta q)^2}, \quad a^0_n = -2\left[p^0 + \frac{2\lambda_n^2}{(\Delta q)^2}\right]. \tag{15}$$

One can show the series (11) converges with respect to m absolutely and uniformly. For this reason we can take into account in the sum over m of the expression (11) only the finite number of terms with any desirable accuracy. It leads to the finite number of the equations (8), (13), (14) and thus the considered problem is getting technically solveable. Now the conditions of nontrivial solution of the finite systems of equations (13), (14) give us two independent equations to determine the threshold value of the light wave electric field. Further, for example, we restrict our consideration only to the terms in the series (11) with $m \leq 2$.

SMALL AMPLITUDES OF THE ANCHORING ENERGY

Denote by λ_1 and λ_1^0 the leasts among the solutions of equation (10) for the cases $|\alpha(y)| \ll W$ and $\alpha(y) = 0$, respectively.

Then, one can restrict oneself at the consideration only to the terms linear in $\Delta\lambda_1 = \lambda_1 - \lambda_1^0$. As a result, from the conditions of nontrivial solution of the systems of equations (13), (14) we can find the next analytical expressions for the threshold value of light wave electric vector:

$$E_{0th} = \sqrt{2}\, E' \left[1 + \frac{\Delta\lambda_1}{\lambda_1^0} - \frac{5}{12} \left(\frac{\lambda_1^0}{\Delta q} \right)^2 \right], \quad \text{if} \quad \Delta q L \gg 2\pi. \qquad (16)$$

$$E_{0th} = E' \left[1 + \frac{\Delta\lambda_1}{\lambda_1^0} + 0.59 \left(\frac{\Delta q}{\lambda_1^0} \right)^2 \right], \quad \text{if} \quad \Delta q L \ll 2\pi. \qquad (17)$$

Here $E' = \lambda_1^0 \left(\frac{2\pi\varepsilon_{\parallel} K}{\varepsilon_a \varepsilon_{\perp}} \right)^{1/2}$ is the threshold value of electric vector at $\Delta q = 0$, $\alpha(y) = 0$ that is at the constant value of both the intensity of light field and anchoring energy.

To obtain the expression for $\Delta\lambda_1$ it is necessary to specify the function $\alpha(y)$. Let suppose the function $\alpha(y)$ to be even. Then the solution (10) contains only the terms with $\cos(m\Delta q y)$, so $D_m^\lambda = 0$ and in the system of equations (9) we must put equal to zero $\tilde{C}_m^\lambda$, $\tilde{D}_m^\lambda$ and α_{ml}^{cs}, α_{ml}^{sc}, α_{ml}^{ss}. In this case the next expression for $\Delta\lambda_1$ follows from the equation (10) :

$$\Delta\lambda_1 = \frac{2\pi K}{W^2 L^2} \sum_{i=0}^{2} \left[\alpha_{ii}^{cc} - \sum_{j=0}^{2} (\alpha_{ij}^{cc2} - \alpha_{ii}^{cc} \alpha_{jj}^{cc}) \right]. \qquad (18)$$

Here we neglected small terms which are cubic in $\alpha(y)$.

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In particular, for $\alpha(y) = \alpha\cos(qy)$ one can get trom (18)

$$\Delta\lambda_1 = -\frac{9}{16}\frac{\alpha^2}{\pi WK}, \text{ if } q = \Delta q \quad \text{and} \quad \Delta\lambda_1 = -\frac{3}{8}\frac{\alpha^2}{\pi WK}, \text{ if } q = 2\Delta q,$$

where the linear in α terms are absent because for the given function $\alpha(y)\sum_{i=0}^{2}\alpha_{ii}^{cc} = 0$.

In the case of the odd function $\alpha(y)$ the expression (18) for $\Delta\lambda_1$ preserves its form but it is necessary to replace α_{ij}^{cc} by α_{ij}^{ss}. For an arbitrary function $\alpha(y)$ (but $|\alpha(y)| \ll W$) the expression for $\Delta\lambda_1$ is similar but much more cumbersome and we do not adduce it here.

GREAT AMPLITUDES OF THE ANCHORING ENERGY

To find E_{0th} in the general case when $|\alpha(y)|$ is of the same order of value as W it is necessary to solve numerically the equation (10) together with the equations (16), (17). Let us consider for example two special cases of the function $\alpha(y)$.

a) $\alpha(y) = \alpha\cos(k\Delta qy + \gamma)$, $WL/K = 10$. The results of calculation of E_{0th} versus light intensity spatial period are shown in Figure 1 for $\alpha = 0.4W, 0.8W$ and $k = 1, 2$. Here it is shown for comparison the curves $E_{0th}(\Delta q)$ at the infinite anchoring energy (notation $W = \infty$) and $\alpha = 0$ (notation $\alpha = 0$). It is necessary to note that the phase difference γ between the waves of modulation of light intensity and anchoring energy does not influen-

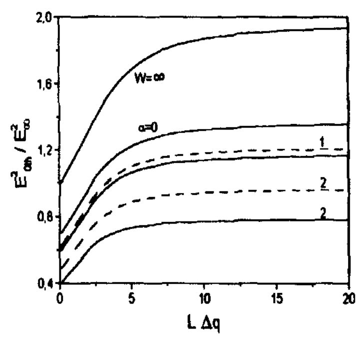

FIGURE 1. $\alpha(y) = \alpha\cos(k\Delta qy + \gamma)$, $\alpha = 0.4W - 1, 0.8W - 2$; $k = 1$ – solid line, $k = 2$ – dashed line; E_∞ is the threshold value at $W = \infty$, $\Delta q = 0$.

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ce the threshold value. But the last depends essentially on the ratio k between the values of the spatial periods of these waves as well as the amplitude α of the anchoring energy modulation. In particular, the threshold takes the minimum values (at the fixed value of α) if the spatial periods of modulation waves of the anchoring energy and light intensity are the same ($k = 1$) (see Figure 1). At the values $k \geq 2$ the threshold increases with increasing k and approaches (due to the averaging of the function $\alpha(y)$ on the spatial period $2\pi/\Delta q$ of the light intensity) to its value at $\alpha(y) = 0$. The threshold value versus $L\Delta q$ for $k = 2$ is shown in Figure 1.

If $k = 1/n$, where integer $n \geq 2$, the period of the anchoring energy modulation contains two or more spatial periods of the light intensity. In this case the threshold does not depend on the amplitude α and coinsides with its value at $\alpha = 0$.

b) Let the $\alpha(y)$ takes the form as in Figure 2. Here $2T_1$ is a period of the anchoring energy modulation, $p \cdot 2T_1 = s \cdot 2\pi/\Delta q$, where s, p are the integers which have no common multipliers.

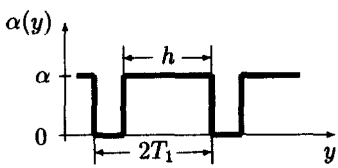

FIGURE 2.

E_{0th} versus h/T_1 is shown in Figure 3 for $\alpha = 0.4W$, $WL/K = 10$ at the different values of s and p. As one could wait, the threshold value E_{0th} increases with increasing of h, since the cell surface region with anchoring energy $W + \alpha$ increases. But it is more interesting the increase of E_{0th} with growth of p at the fixed values of s and h (see Figure 3a). As p increases the distribution of the anchoring energy inhomogeneity on the problem period (or on the longitudinal dimension of nematic cell) becomes more uniform. Note that the threshold value leaves unchanged (at change of p) at some values of h/T_1. At these h/T_1 the summary length of the regions with the anchoring energy $W+\alpha$ equals the whole number of light intensity spatial periods so all possible values of light intensity correspond to the anchoring energy $W + \alpha$ and the variation of p does not change this correspondence.

In Figure 3b E_{0th} versus h/T_1 is shown for different values of s at $p = 1$. One can see the increase of E_{0th} with s (the number of light intensity spatial periods which is placed on the problem period). In this case the increase of E_{0th} is connected with the growth of degree of light intensity averaging on the problem period. At some h/T_1 (see arguments above) the E_{0th} does not change its value.

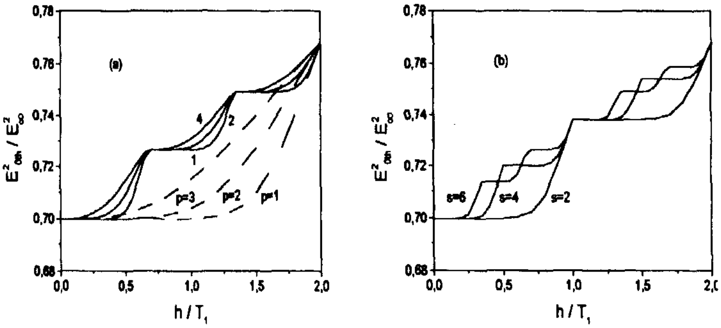

FIGURE 3.
a) $s = 1$ – dashed line, $p = 1, 2, 3$; $s = 3$ – solid line, $p = 1, 2, 4$.
b) $p = 1$, $s = 2, 4, 6$.

In the real conditions one can wait the periods $2\pi/\Delta q$ and $2T_1$ to be incommensurable. In our model it means that numbers p and s approach infinity. As s increases the number of knot points (see Figure 3b) on the curve $E_{0th}(h)$ increases as well. At that time the parts of the curve $E_{0th}(h)$ between the knot points approach the straight lines as p increases. So the $E_{0th}(h)$ takes the form of the enveloped curve $E_{0th}^{ef}(h)$. Besides, evidently (see Figure 3b) that at $s \gg 1$ the dependence of E_{0th} on p becomes less important. Therefore at $p \gg 1$ E_{0th} does not depend practically on p. It means that the distribution of the anchoring energy value $W + \alpha$ on the longitudinal dimension of nematic cell does not influence the E_{0th} if the summary length of the $(W + \alpha)$ region preserves. Then one can determine the effective value of $h/(2T_1)$ necessary to calculate the E_{0th} as hp/L_0, were hp is the summary length of the $(W + \alpha)$ regions, L_0 is the

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length of nematic cell. At last it is necessary to note that the curve $E^{ef}_{0th}(h)$ is close to the straight line. It can be usefull for estimation of E_{0th} at different values of h.

CONCLUSIONS

The obtained system of algebraic equations allows to find the light induced Frederiks transition threshold in nematic cell with the director inhomogeneous anchoring energy. At small amplitudes of the anchoring energy the analytical expressions for threshold are obtained. At great anchoring energy amplitudes the numerical solution of equations is necessary. In this case the dependence of the threshold on the parameters of the anchoring energy and light field is illustrated for some types of the anchoring energy inhomogeneity.

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